The Radius of the Electron, and the Nuclear Structure of Atoms

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I. *The Radius of the Electron, and the Nuclear Structure of Atoms.* By *Prof.* J. W. NICHOLSON, *M.A., D.Sc., F.R.S.*

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THE present note is intended rather to make a suggestion than to formulate any definite theory of the structure of the nucleus of an atom according to the model at present found necessary in order to interpret such phenomena as radioactivity, atomic number, and scattering of charged particles by atoms. The electron is usually regarded as a kind of globule of electricity with a definite radius, and as the nuclei of the more complex atoms must, from certain considerations, be supposed to contain electrons, and at the same time preserve their minute size, a difficulty is encountered unless we may suppose that electrons and positive charges can actually in some way inter-penetrate each other and occupy the same space. Some means of removing the finite radius of an electron, and with it all discontinuity at a prescribed surface, is manifestly desirable. On theories such as that of Lorentz, the electron, a sphere when at rest, is deformed when in motion, but in all cases in which hypotheses as to the inner structure and internal equilibrium of an electron are introduced, it has been given, when at rest, this definite "radius," marking off a distinct region from the aether, and involving a discontinuity of some form at the boundary. However great the departure of this view from the more natural intuitions or prejudices of those who regard the electron as a structure built out of aether, it has been of great service at many points. Especially, in the hands of Lorentz and others, it has led to a conception of the variation of the mass of an electron with its speed, which is in undoubted agreement with careful experiments, and must contain a large substratum of truth. Such considerations involve the existence of a linear constant which is the same for every electron and is usually regarded as a "radius." A similar constant is, of course, necessary for the elementary positive charge.

If the terminology which makes use of an aether, out of which the elementary charges are constructed as regions of strain, is adopted, it would seem more natural that such line-constants should be constants with some significance through out the whole aether, rather than constants which only come into being when the aether is strained into the form of matter. The aether may, in fact, be in some manner cellular, with these linear magnitudes involved in the specification of the cells, and thereby in any strained structure composed from them. In this Paper we accordingly make a tentative suggestion towards this form of interpretation of the line-constants by regarding the electron as a state of strain which is for practical purposes concentrated at its "centre," rapidly diminishing outwards from this point according to some very convergent law, which involves a line-constant in its specification. It is found that no important difference is made in the mutual reactions of electrons except at distances comparable with their "radii," and that if the strains are regarded as capable of superposition, inter-penetration is readily possible without the introduction of indefinitely large forces between the components. A form in which neutral doublets could exist as a part of nuclear structure also becomes evident.

The mathematical treatment, on the basis of a simple exponential law of attenuation of the strain, is only an illustration developed for purposes of clearness. If the suggestion were to correspond in any way with reality, no phenomena at present available could give a clue to the actual law. An elementary argument on the basis of physical dimensions, however, is sufficient to show that any other law would only lead to certain differences in numerical coefficients.

Sir Joseph Larmor alone appears not to be definitely bound to the point of view of the finite electron. For the purposes of his theory,* an electron is a type of singularity, made up of aether in a peculiar state of strain, which needs no more precise mathematical definition of the state of strain than is implied in the relation

$$\iint (lf + mg + nh)\,ds = e,$$

or, the surface integral of aethereal polarisation over any surface surrounding one electron is $4\pi e$. This serves as a definition of e. In another form, an electron is a region in which the divergence of the aethereal polarisation is not zero. From this point of view, an electron might have no boundary, and the singularity could be confined effectively to a very small region by choosing a distribution of electrical density ρ, defined by

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* "Æther and Matter," Camb. Univ. Press, 7900, pp. 86-90.

$$\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}+\frac{\partial h}{\partial z}=\rho,$$

following any rapidly convergent law of variation, for example, $\rho=\epsilon^{-\lambda r}$, where r is distance from a point. The departure of the surface integral from $4\pi e$ would be inappreciable within a few diameters of the electron, if a diameter is defined as a length comparable with λ^{-1}.

It is noteworthy that Lorentz, whose electron, of those investigated in detail, alone seems to give a reasonable description of those physical phenomena for which the nature of the electron is important, should have found it necessary to impose a contraction on the moving electron of like magnitude with the contraction of all the "interspaces" between electrons, found earlier to the second order by Larmor, by an analysis which is equally applicable to all orders. This equality of the contraction, whether regarded from the point of view of the Principle of Relativity or not, seems in itself to imply a continuity of the electron with the rest of space—an aethereal structure for the electron with no defined boundary. A subsequent brief survey of the consequences of the exponential formula already suggested, as a mere illustrative case, will indicate the bearing of this remark.

But a length is necessarily associated with an electron, as is even apparent from the existence of its electromagnetic mass, with its necessary dimensions. Such a length in the exponential case would be given by λ^{-1}. Whatever the physical significance of this length, it is evident that a view which attaches to it the same significance for all parts of an infinite aether is more acceptable on some grounds than one which relates it to a definite region only round the "centre" of the electron. The endowment of the whole aether with such a linear constant is ultimately equivalent to an endowment with some form of structure. Such a possibility of aethereal structure might ultimately solve many difficulties. It appears to be demanded by the recent quantum or unit theory of energy—provided that the view of an aether is retained—and it would involve an equality of λ for all electrons, since λ would no longer be a property of the electron, but of the whole aether. There is evidence that e, the charge of an electron, bears a relation to Planck's quantum constant h. These brief indications will suffice to show that we can find a means of removing the boundary of an electron while retaining all the analysis of Lorentz, so that a reconciliation of the quantum theory with the necessary properties of the electron, and, in fact, a deduction of the one from the other, is a possible hope for the future. The analysis of the bounded electron, which becomes spheroidal when in motion, then appears as a convenient mathematical substitute for the more complete analysis of an electron of infinite extent, consisting of a state of strain impressed on a structural aether, but a strain of such a mathematical form that it is confined for physical purposes in a certain region of minute extent, comparable in diameter with a linear magnitude involved in the structure of the aethereal cells.

When, in the ordinary view, the radius of an electron is a, the usual formula for the mass of the electron, of electromagnetic origin, is

$$m_0 = \frac{2}{3}\frac{e^2}{ac^2}$$

for speeds small in comparison with c, the velocity of light in free aether. In the present note, we shall not be concerned with a higher approximation on account of the comparatively small velocities which it is necessary to ascribe to the electrons in the model atoms which are found capable of giving some account of such phenomena as spectral series.

We proceed to a brief survey of the illustrative case of a more continuous electron, already indicated in the preceding pages. It is not claimed that this is more than a mere illustration of the possibilities of such a theory.

Let us define an electron-centre as an origin around which there is a symmetrical distribution of polarisation satisfying

$$\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = \rho = A\varepsilon^{-\lambda r},$$

where λ is a constant, the reciprocal of a certain length fixed by the aethereal structure. In polar co-ordinates—

$$\frac{1}{r^2}\frac{\partial}{\partial r}(fr^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(g\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial h}{\partial\phi} = A\varepsilon^{-\lambda r},$$

where (fgh) are now radial and transversal components. In case of a dependence only on r,

$$\frac{\partial}{\partial r}(fr^2) = Ar^2\varepsilon^{-\lambda r},$$

and we may write $g = h = o$.

Thus $$fr^2=-A\left\{\frac{r^2}{\lambda}+\frac{2r}{\lambda^2}+\frac{2}{\lambda^3}\right\}\varepsilon^{-\lambda r}+\text{constant}.$$

With the constant equal to $2A/\lambda^3$, f is finite at the origin, and in fact zero, so that

$$f=-A\left\{\frac{1}{\lambda}+\frac{2}{r\lambda^2}+\frac{2}{r^2\lambda^3}\right\}\varepsilon^{-\lambda r}+\frac{2A}{\lambda^3 r^2},$$

and at a great distance, the intensity of electric force follows the law of inverse square.

The total charge of the electron being e,

$$e=\iiint\rho\, dx\, dy\, dz$$

taken throughout space, or

$$\begin{aligned}e&=A\int_0^\infty\int_0^{2\pi}\int_0^\pi\varepsilon^{-\lambda r}r^2\sin\theta\, dr\, d\phi\, d\theta\\&=4\pi A\int_0^\infty r^2\varepsilon^{-\lambda r}dr,\\&=8\pi A/\lambda^3,\end{aligned}$$

so that at a sufficient distance

$$f=\frac{e}{4\pi r^2},$$

and the electric force is e/r^2. The true polarisation would be

$$f=\frac{e}{4\pi r^2}-\frac{e}{8\pi}\left\{\frac{2}{r^2}+\frac{2\lambda}{r}+\lambda^2\right\}\varepsilon^{-\lambda r}.$$

The electrostatic energy in the field becomes

$$W=\frac{1}{8\pi}\iiint(4\pi f)^2 r^2\, d\omega\, dr.$$

or, after considerable reduction, at various stages

$$\begin{aligned}W&=\frac{1}{2}e^2\int_0^\infty dr\left\{\frac{1}{r}-\left(\frac{1}{r}+\lambda+\frac{\lambda^2 r}{2}\right)\varepsilon^{-\lambda r}\right\}^2.\\&=\frac{\lambda e^2}{32}-\frac{\lambda e^2}{8}+\frac{1}{2}e^2\int_0^\infty\frac{dr}{r^2}\left\{1-(1+\lambda r)\varepsilon^{-\lambda r}\right\}^2.\\&=-\frac{3}{32}\lambda e^2+\lambda^2e^2\int_0^\infty dr\,\varepsilon^{-\lambda r}\left\{1-(1+\lambda r)\varepsilon^{-\lambda r}\right\}\end{aligned}$$

(integrating by parts)

$$=-\frac{3}{32}\lambda e^2+\frac{\lambda e^2}{4}=\frac{5}{32}\lambda e^2.$$

A more difficult problem is the determination of the mutual energy of two electrons occupying the aether at the same time, so that their fields are superposed. From the solution, a knowledge of a change in the law of force between the electrons at distances comparable with λ, can be obtained.

If for convenience we write

$$rf(r) = \frac{1}{r} - \left(\frac{1}{r} + \lambda + \frac{\lambda^2 r}{2}\right)\epsilon^{-\lambda r},$$

and (r, r') are distances measured from the electrons, which are in the z axis, the portion of the field energy which is mutual is—

$$W_1 = \frac{e^2}{4\pi}\iiint f(r)f(r')d\tau,$$

where $d\tau$ is an element of volume, or

$$W_1 = e^2\int_0^\infty\int_0^\pi f(r)f(r')r^2 \sin\theta\, dr\, d\theta,$$

where $r' = \sqrt{r^2 + z^2 - 2rz\cos\theta}$,

z being the distance between the electron-centres.

We can proceed otherwise as follows: The electric forces (PQR), $(P'Q'R')$ of the electrons are derived from potentials V, V', and the mutual energy in any region is

$$\frac{1}{4\pi}\iiint\left(\frac{\partial V}{\partial x}\frac{\partial V'}{\partial x} + \frac{\partial V}{\partial y}\frac{\partial V'}{\partial y} + \frac{\partial V}{\partial z}\frac{\partial V'}{\partial z}\right)d\tau$$

$$= -\frac{1}{4\pi}\iint V'\frac{\partial V}{\partial n}dS - \frac{1}{4\pi}\iiint V'\nabla^2 V d\tau.$$

by the usual notation of Green's theorem, and the surface integral is negligible over the infinite boundary of the whole field, where V, V' are each of order $1/r$. Moreover, $\nabla^2 V' = 4\pi\rho'$, and in the usual way,

$$W_1 = \iiint V\rho' d\tau,$$

where the integral is taken over all space. This becomes

$$W_1 = \frac{e\lambda^3}{8\pi}\iiint V\epsilon^{-\lambda r}r^2 dr\, d\omega$$

$$= \frac{1}{4}e\lambda^3\int_0^\infty\int_0^\pi Vr^2 \sin\theta\,\epsilon^{-\lambda r}dr\, d\theta,$$

the origin being at the first electron, and $d\omega$ an element of solid angle. In order to obtain V, we note that it vanishes at infinity, and that $-\partial V/\partial r$ is the electric intensity due to an electron situated at the origin. Thus

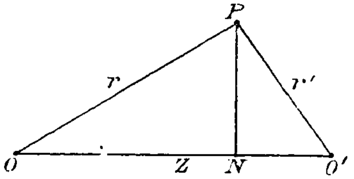
$$-\frac{\partial V}{\partial r}=\frac{e}{r^2}-e\left(\frac{1}{r^2}+\frac{\lambda}{r}+\frac{\lambda^2}{2}\right)e^{-\lambda r}$$

$$V=\int_r^\infty\frac{e\,dr}{r^2}-\int_r^\infty e\left(\frac{1}{r^2}+\frac{\lambda}{r}+\frac{\lambda^2}{2}\right)e^{-\lambda r}dr$$

$$=\frac{e}{r}-\frac{e}{r}\left(1+\frac{\lambda r}{2}\right)e^{-\lambda r}.$$

The potential at the centre of the electron is finite and equal to $\frac{1}{2}e\lambda$.

It is more convenient to use bipolar co-ordinates, for we must avoid, in the integrations, attaching a negative sign to r or r'.



Taking the centres O, O' of the electrons as origins, then any point, P, or (r, r') can equally be defined by the values of $r \pm r'$. Moreover, the curves $r \pm r' =$ constant are the set of confocal ellipses and hyperbolas in any plane through O, O', with these points as foci. If $c = \frac{1}{2}z$, and $(\xi\eta)$ are the elliptic co-ordinates defined by

$$x = c\cosh\xi\cos\eta, \qquad y = c\sinh\xi\sin\eta,$$

then we readily find

$$(r, r') = c(\cosh\xi \pm \cos\eta),$$

which are essentially positive, since the minimum value of $\cosh\xi$ is unity. The semi-major axis of the ellipse through any point is, of course, $c\cosh\xi$, or $\frac{r+r'}{2}$.

Write $r + r' = \rho$, $r - r' = \sigma$, when r is greater than r'. Then ρ and σ are orthogonal co-ordinates in space, and their directions of increase are those of (ξ, η), or normal to the ellipse and hyperbola through any point in the plane of O, O'. The elements ds_1 and ds_2 of length along ρ and σ increasing are then given by

$$ds_1 = d\xi\left\{\left(\frac{\partial x}{\partial \xi}\right)^2+\left(\frac{\partial y}{\partial \xi}\right)^2\right\}^{\frac{1}{2}} = c\sqrt{\cosh^2\xi - \cos^2\eta}\, d\xi$$

$$ds_2 = d\eta\left\{\left(\frac{\partial x}{\partial \eta}\right)^2+\left(\frac{\partial y}{\partial \eta}\right)^2\right\}^{\frac{1}{2}} = c\sqrt{\cosh^2\xi - \cos^2\eta}\, d\eta$$

or
$$ds_1 ds_2 = c^2 d\xi d\eta(\cosh^2\xi - \cos^2\eta).$$

The third element may be defined by a rotation $d\phi$ of the line PN round the axis O. O', and has a length

$$ds_3 = PNd\phi = c\sinh\xi\sin\eta d\phi.$$

Thus the volume of a rectangular element of space at the point P is

$$d\tau = c^3 d\xi d\eta d\phi \sinh\xi \sin\eta(\cosh^2\xi - \cos^2\eta).$$

But
$$2c\cosh\xi = r+r' = \rho, \quad 2c\sinh\xi d\xi = d\rho$$
$$2c\cos\eta = r - r' = \sigma, \quad 2c\sin\eta d\eta = -d\sigma,$$

so that
$$d\tau = \frac{\rho^2 - \sigma^2}{16c} d\phi d\rho d\sigma.$$

and the mutual potential energy becomes

$$W_1 = \iiint (V\rho') \cdot \frac{\rho^2 - \sigma^2}{8a} d\phi d\rho d\sigma.$$

Taking account of both cases, $r < r'$, $r' < r$, we find that this formula gives W_1 when taken between the limits

$$\phi = 0 \text{ to } 2\pi$$
$$\rho = a \text{ to } \infty$$
$$\sigma = -a \text{ to } a.$$

Performing one integration

$$W_1 = \frac{\pi}{4a}\int_a^\infty d\rho\int_{-a}^{a} d\sigma(V\rho')(\rho^2 - \sigma^2)$$

in a form readily applicable to electrons of any law of density. For the present case

$$\rho' = \frac{e\lambda^3}{8\pi}\varepsilon^{-\lambda r'} = \frac{e\lambda^3}{8\pi}\varepsilon^{-\frac{\lambda}{2}(\rho-\sigma)}$$

$$V = \frac{e}{r}\left\{1 - \left(1+\frac{\lambda r}{2}\right)\varepsilon^{-\lambda r}\right\}$$

$$= \frac{2e}{\rho+\sigma}\left\{1 - \left[1+\frac{\lambda}{4}(\rho+\sigma)\right]\varepsilon^{-\frac{\lambda}{2}(\rho+\sigma)}\right\},$$

and thus

$$W_1=\frac{e^2\lambda^3}{16z}\int_z^\infty\int_{-z}^{z}(\rho-\sigma)d\rho d\sigma\left\{1-\frac{\lambda(\rho+\sigma)}{4}\varepsilon^{-\frac{\lambda}{2}(\rho+\sigma)}\right\}\varepsilon^{-\frac{\lambda}{2}(\rho-\sigma)}$$

$$=\frac{e^2\lambda^3}{16z}(I_1+I_2).$$

where

$$I_1=\int_z^\infty\int_{-z}^{z}d\rho d\sigma(\rho-\sigma)\varepsilon^{-\frac{\lambda}{2}(\rho-\sigma)}$$

$$I_2=-\frac{\lambda}{4}\int_z^\infty\int_{-z}^{z}d\rho d\sigma(\rho^2-\sigma^2)\varepsilon^{-\lambda\rho}.$$

Now

$$I_1=\varepsilon^{-\frac{\lambda z}{2}}\int_0^\infty\int_{-z}^{z}du d\sigma(z+u-\sigma)\varepsilon^{\frac{\lambda}{2}(\sigma-u)}\qquad(\rho=u+z)$$

$$=\varepsilon^{-\frac{\lambda z}{2}}\int_{-z}^{z}d\sigma\left\{\frac{2}{\lambda}(z-\sigma)+\frac{4}{\lambda^2}\right\}\varepsilon^{\frac{\lambda}{2}\sigma}$$

$$=\frac{2}{\lambda}\varepsilon^{-\frac{\lambda z}{2}}\left[\left\{\left(z-\sigma+\frac{2}{\lambda}\right)\frac{2}{\lambda}+\frac{4}{\lambda^2}\right\}\varepsilon^{\frac{\lambda}{2}\sigma}\right]_{-z}^{z}$$

$$=\frac{16}{\lambda^3}-\frac{4}{\lambda^2}\left(2z+\frac{4}{\lambda}\right)\varepsilon^{-\lambda z}$$

$$I_2=-\frac{\lambda}{2}\int_z^\infty d\rho\left(\rho^2 z-\frac{z^3}{3}\right)\varepsilon^{-\lambda\rho}$$

$$=-\frac{\lambda}{2}\left[\left(\frac{z^3}{3\lambda}-\frac{\rho^2 z}{\lambda}-\frac{2\rho z}{\lambda^2}-\frac{2z}{\lambda^3}\right)\varepsilon^{-\lambda\rho}\right]_z^\infty$$

$$=-\frac{\lambda}{2}\left(\frac{2z^3}{3\lambda}+\frac{2z^2}{\lambda^2}+\frac{2z}{\lambda^3}\right)\varepsilon^{-\lambda z}$$

$$=-\frac{z}{\lambda^2}\left(1+\lambda z+\frac{\lambda^2z^2}{3}\right)\varepsilon^{-\lambda z}.$$

And finally

$$W_1=\frac{e^2}{z}-\frac{e^2}{4z}(4+2\lambda z)\varepsilon^{-\lambda z}-\frac{e^2\lambda}{16}\left(1+\lambda z+\frac{\lambda^2z^2}{3}\right)\varepsilon^{-\lambda z}$$

is the mutual potential energy of the electrons, as a function of their distance z apart.

The force tending to increase z is $F=-\partial W_1/\partial z$, or

$$F=\frac{e^2}{z^2}-\frac{e^2\lambda^3z}{48}(1+\lambda z)\varepsilon^{-\lambda z}-\frac{e^2}{z^2}\left(1+\lambda z+\frac{1}{2}\lambda^2z^2\right)\varepsilon^{-\lambda z}$$

becoming e^2/z^2 at a great distance, as in the ordinary electron.

But at small distances the force does not tend to infinity, but to zero. We find

$$F\varepsilon^{\lambda z} = \frac{e^2}{z^2}\left(\varepsilon^{\lambda z} - 1 - \lambda z - \frac{1}{2}\lambda^2 z^2\right) - \frac{e^2\lambda^3 z}{48}(1+\lambda z)$$
$$+\frac{e^2\lambda^3 z}{18}\left(1+\frac{\lambda z}{7}\right)$$

when z is small in comparison with λ^{-1}. The force vanishes when the centres coincide, forming an electron of strength $2e$, as we should expect.

The exponentials are, for ordinary purposes, negligible even when z is a few multiples of λ^{-1}, which we may regard as the "radius" of the electron.

The electron $2e$ is essentially unstable, and e itself must be regarded as a constant of the aethereal structure, like λ. There is a probable relation between e and Planck's unit, as stated already.

Even when z is comparable with λ^{-1}, the force sinks to a small fraction of its value as given by the usual formula. For example, if $z = 2/\lambda$, the force is only $0{\cdot}05e^2/z^2$.

This tendency of the force to vanish when the electrons are very close is, of course, applicable to electrons of opposite sign. But the inertia of the positive electron must be large, and this involves a large value of λ. We can show without difficulty that the inertia is proportional to λe^2 for slow speeds, and thus λ for the positive electron, if such exists, must be of the order of 1,000 times the value of λ for a negative electron.

But the practical evanescence of the force will remain, and there is even the possibility that the sign of the force may change, with a suitable law of density. Thus a positive and negative electron would not necessarily rush together and annihilate each other, but would form a doublet, whose length would be comparable with the radius of a single electron. Even if the force did not change sign, so that the positive and negative elements had no position of relative equilibrium, an oscillation about each other in simple harmonic motion is possible. For example, if the values of λ were equal, the electrons being of the above type, the equation for the distance z between them would be

$$\frac{m\ddot{z}}{2} = -\frac{7e^2\lambda^3 z}{48},$$

where m is the mass of either. Since m is of order λe^2, the system would emit a wave-length comparable with the "radius" of an electron.

If the doublet is endowed with a rotation, it can preserve a constant length, and the present investigation is given merely as an illustration of the possibility of such doublets.

The view that two aethereal structures can exist in this way without deformation in presence of each other, and simultaneously occupying the whole aether, is, of course, difficult. But the difficulty is no greater than that of postulating the ordinary bounded electron, each of whose parts must repel each other. If actual deformation takes place, it is not apparently possible to find a basis of calculation without further hypotheses incapable of verification, so that the present suggestion of the possible nature and existence of doublets is sufficient for the purpose in view. It is, perhaps, worthy of remark that the Lorentz formula for mass as a function of velocity can be obtained for this type of electron, with λ^{-1} substituted for the radius. The whole distribution of density ρ may be treated as in the principle of relativity. It does not seem necessary to give the complete analysis.

From this point of view a neutral doublet could consist of a distribution of density of the form

$$\rho = \frac{e}{8\pi}(\lambda_1^3 e^{-\lambda_1 r} - \lambda_2^3 e^{-\lambda_2 r})$$

around the origin as centre. The total charge is zero when integrated throughout space, and the density and electric force are confined to a region round the origin whose radius is of order λ_1^{-1} $(\lambda_1 < \lambda_2)$ almost precisely. The system behaves like a pair of charges $\pm e$ at the origin, of radii λ_1^{-1}, λ_2^{-1}, which can be separated by the application in a suitable form of an amount of energy of order $\lambda_1 e^2$.

Such a type of doublet, whose charges were any multiple of that of an electron, might form a component of a complex atomic nucleus, and radioactivity would then occur when it was partially dissolved into component α and β particles. It would appear to be necessary to consider a theory of this type, in view of the extreme smallness of the nuclei of atoms, as determined by Rutherford and others, which nevertheless possess the property of evolving large numbers of α and β particles, the latter being of the same order of size as the nucleus itself.

In again laying emphasis on the fact that the present note is only intended to be a suggestion towards a point of view. rather than a development of the view which could serve no useful purpose at the present stage—Sir Ernest Rutherford has proposed to leave nuclear structure to the next generation—it is to be noticed that the difficulties inherent in the permanent fixation of these types of strain, capable of superposition, are not greater than those involved in the internal equilibrium of the mutually repelling parts of the ordinary bounded electron.

ABSTRACT.

The electron is usually regarded as a globule of electricity with a definite radius. This conception has proved valuable, but involves difficulties in connection with the nuclear structure of complex atoms. On the view that the electron consists of a region of strain in the æther such line constants should have some significance throughout the whole æther; which may, in fact, be in some manner cellular with these linear magnitudes involved in the specification of the cells, and therefore in any strained structure composed of them.

The electron would be regarded as a state of strain which for practical purposes is concentrated at its centre. rapidly diminishing outwards according to some very convergent law involving some line constant in its specification. By way of illustration the idea is worked out mathematically on the assumption that the strain varies as $e^{-\lambda r}$, on which hypothesis λ^{-1} is the "radius." It can be shown that the Lorentz formula for mass as a function of velocity can be obtained for this type of electron. The charge on the electron is regarded as a fundamental property of the æther. and is related to Planck's constant h.

DISCUSSION.

Dr. H. S. ALLEN: There can be little doubt of the existence of a relation, referred to by Prof. Nicholson, between Planck's constant h and the charge of an electron e. The relation suggested by Lewis and Adams may be written—

$$ch \sqrt[3]{\frac{8\pi^5}{15}(4\pi e)^2},$$

where c is the velocity of light. Taking Millikan's latest value for e ($4{\cdot}774 \times 10^{-10}$) and $c = 3 \times 10^{10}$, we find $h = 6{\cdot}558 \times 10^{-27}$. From his photoelectric experiments Millikan found $h = 6{\cdot}57 \times 10^{-27}$ within about 0·5 per cent., and in his latest table of fundamental constants he gives $h = (6{\cdot}547 \pm 0{\cdot}013) \times 10^{-27}$. Thus the agreement is within the limits of experimental error. All the principal radiation constants can be expressed in terms of e. The curious numerical relations between the primary constants of physics, to which attention was directed in my Paper read before the Society in 1915, depend upon the above formula connecting h and e. On the lines suggested in Prof. Nicholson's Paper it would seem as if most, if not all, of the important constants of nature may be referred to some fundamental property of the æther.

Sir OLIVER LODGE (communicated): I am much interested in Prof. Nicholson's ingenious plan for doing away with the definite boundary of an electron, and devising a mathematical scheme which shall enable us to regard it as a point-centre of strain decreasing exponentially in every direction without limit, so that the linear dimension associated with it shall be—like many time-constants—the distance at which the density is reduced to $\frac{1}{e}$th of what it is at the centre. This plan, if it can be developed properly, seems to get over many of the difficulties about the coherence of parts of a charge, and about the extraordinary properties of a nucleus, which though, from some points of view, an extremely small and highly-charged unit, yet necessarily has a complexity which enables it to be disintegrated and fired off in fragments. The ready permeability or inter-penetrability without destruction, of Prof. Nicholson's conception of an electric unit, seems likely to diminish the difficulty of conceiving such a nucleus; and on the whole his suggestions seem to me helpful and valuable. I do not feel justified in saying more at the present time.

Prof. Nicholson, in reply, said it was of interest to see that Millikan's final value of h was practically equal to the first value that had been obtained for that constant.